

EXERCISES FUCHSIAN DIFFERENTIAL EQUATIONS FALL 2022

Herwig HAUSER

21 Find the (unique) third order Euler operator E of shift 0 with local exponents 1, 4, 9 (necessarily of multiplicities 1). Then determine the image of $\mathcal{F} = x\mathbb{C}[[x]]$ and $\mathcal{G} = x\mathbb{C}[[x]] \oplus x\mathbb{C}[[x]]z$ under E , respectively its extension \underline{E} .

22 Let E be the second order Euler operator with local exponents 1 and $3/2$. Show directly that for $\mathcal{F} = x\mathbb{C}[[x]] \oplus x\sqrt{x}\mathbb{C}[[x]]$ one gets $E(\mathcal{F}) = x\mathcal{F}$.

23 (a) Determine a differential operator $L = L_0 + L_1$ for Euler operators L_1 and L_2 of shift 0 and 1, such that 0 and -2 are the local exponents of L (say, of L_0) at 0, and such that, for $\mathcal{F} = \mathbb{C}[[x]] + x^{-2}\mathbb{C}[[x]] = x^{-2}\mathbb{C}[[x]]$, one has

$$L_1(\mathcal{F}) \subset L_0(\mathcal{F}) \subsetneq x\mathcal{F}.$$

(b) Then try to solve $Ly = 0$ in \mathcal{F} .

24 Find a linear differential equation satisfied by

(a) $y(x) = \exp \frac{1}{x}$;

(c) $y(x) = \log(x)^2$;

(d) $y(x) = \exp \frac{1}{x} \log(x)^2$;

(e) $y(x) = x \exp \frac{1}{x} \log(x)^2$.