EXERCISES FUCHSIAN DIFFERENTIAL EQUATIONS FALL 2022

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9 Try to solve the following equations

$$x^3y'' + y = 0,$$

$$x^2y'' + y = 0,$$

$$xy'' + y = 0,$$

$$y'' + y = 0,$$

$$y'' + xy = 0.$$

10 Find an Euler differential equation $Ey = 0$ which has at 0 local exponents 1, 2, and 3 of multiplicities 1, 1, and 2 respectively. Then solve it.

11 Let the derivation $\underline{\partial}$ on $K((x))[z]$ be defined by $\underline{\partial}x = 1$ and $\underline{\partial}z = \frac{1}{x}$, with $K((x))$ the field of formal Laurent series. Prove that

$$\underline{\partial}^j(x^t z^k) = [t^{\underline{j}} z^j + (t^{\underline{j}})' k z^{j-1} + \ldots + \tfrac{1}{j!}(t^{\underline{j}})^{(j)} k^{\underline{j}}] \cdot x^{t-j} z^{k-j}.$$

12 Determine all singularities (in $\mathbb{P}^1_{\mathbb{C}}$) of the hypergeometric equation

$$x(x-1)y'' + +[c - (a+b+1)x]y' - aby = 0,$$

for $a, b, c \in \mathbb{Q}$, together with its local exponents.